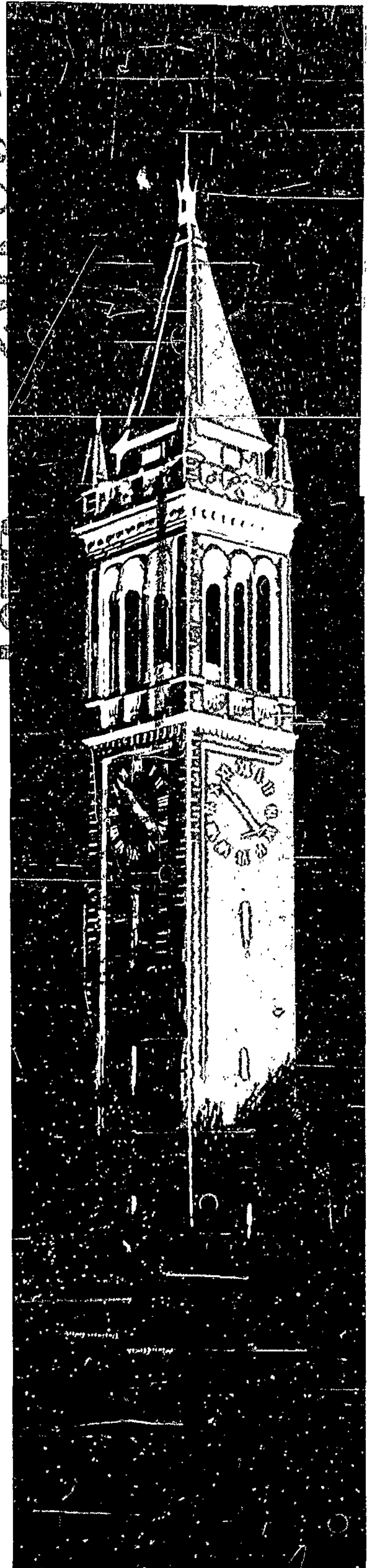

Correlation Processes in Antenna Arrays Part II

by

I. W. Linder

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I. INTRODUCTION

This report is the third in a series reporting investigations into the use of statistical correlation techniques in combining the voltages induced on the elements of a receiving antenna array. Previous reports[1,2] have dealt with the use of correlation methods in analyzing a normal linear array and with the employment of nonlinear processes in the antenna circuitry. The present report continues the analysis of the nonlinear processes in these arrays when both signal and distributed noise sources are present.

The signal and noise sources induce voltages on the elements of a linear antenna array. These voltages are then combined, using summation and multiplication methods, to produce a single array output voltage. When two voltages are multiplied, and then are passed through an averaging device, the resulting voltage corresponds to the statistical correlation coefficient:

$$R(\tau) = \lim_{T \longrightarrow \infty} \frac{1}{2T} \int_{-T}^{T} v(t)\, v\,(t+\tau)\, dt.$$

For this reason, analysis of antenna arrays in which multiplication occurs is facilitated by the powerful tools of probability theory.

In this report the effect of distributed noise is analyzed, resulting in an expression for element spacing which will minimize the undesired noise voltage induced on the array. Channel capacities of linear and of nonlinear arrays are then compared. These results are employed to construct nonlinear arrays which will optimize the signal-to-noise ratio or will provide a minimum beamwidth in the antenna directional pattern.

II. EFFECT OF DISTRIBUTED NOISE ON THE NONLINEAR ANTENNA SYSTEM

The following assumptions will be made:

1. uniform distribution of independent random noise sources;
2. noise sources in an element $d\Omega$ induce a noise power

$\eta d\Omega$ watts/cps in an isotropic antenna element;

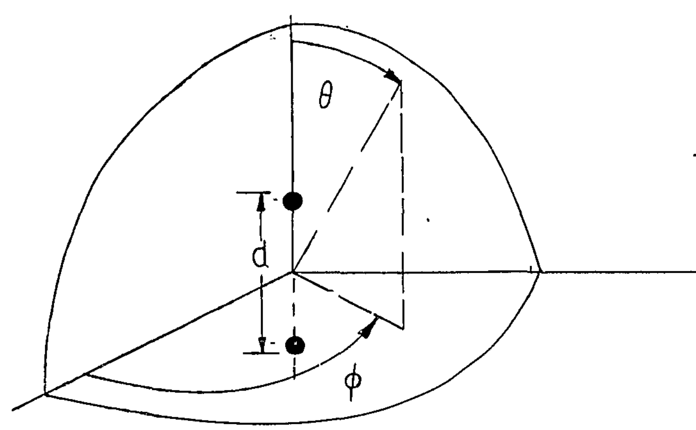
3. antenna element and associated circuitry have a rectangular passband of bandwidth B, center frequency f_o, with $f_o >> B$.

Then the autocorrelation function of the noise voltage induced on an antenna element is

$$\psi_{11}(s) = \overline{x_1(t)x_1(t+s)} = 4\pi B\eta \frac{\sin\pi Bs}{\pi Bs} \cos \omega_o s$$
$$= R_n(o) \frac{\sin\pi Bs}{\pi Bs} \cos \omega_o s$$

where $R_n(0)$ represents the average noise power.

With two identical isotropic elements, separated by a distance d, the voltage induced on one element by the noise sources in an angular area $d\Omega$ differs from that induced on the second antenna element only by a delay time τ.



$$\tau = \frac{d}{c} \cos \theta$$

The same average power, $R_n(0)$, is induced on each element. The autocorrelation coefficient of the noise voltage on each element is the same.

The cross-correlation coefficient becomes:

$$\psi_{12}(s) = \overline{k_1(t)x_2(t+s)} = \overline{x_1(t)x_1(t+\tau+s)}$$

$$= \int_0^{2\pi}\int_0^{\pi} B\eta \frac{\sin \pi B(\tau + s)}{\pi B(\tau+s)} \cos \omega_o(\tau+s) \sin\theta \, d\theta \, d\phi$$

$$= 2\pi B \int_0^{\pi} \frac{\sin \pi B(\frac{d}{c}\cos\theta + s)}{(\pi B \frac{d}{c}\cos\theta + s)} \cos\omega_o (\frac{d}{c}\cos\theta + s) \sin\theta \, d\theta$$

This can be integrated to yield:

$$\psi_{12}(s) = \frac{1}{4} R_n(0) \frac{c}{\pi Bd} \{ Si[(k+1)(\frac{\pi Bd}{c} + \pi Bs)] - Si[(k+1)(\frac{-\pi Bd}{c} + \pi Bs)]$$

$$- Si[(k-1)(\frac{\pi Bd}{c} + \pi Bs)] + Si[(k-1)(\frac{-\pi Bd}{c} + \pi Bs)]\}$$

where $k = \frac{\omega_o}{\pi 8}$ and $Si(x)$ is the sine integral.

By the symmetry of the integrand and of the range of integration it is evident that

$$\psi_{21}(s) = x_2(t)x_1(t+s) = x_1(t+\tau)x_1(t+s) = \psi_{12}(s)$$

Under the usual conditions of $f_o >> B$, and for other than very close element spacings, this expression for the cross-correlation coefficient can be considerably simplified. Under these conditions, approximate $Si(x)$ by $\frac{\pi}{2} - \frac{\cos x}{x}$, and, after some manipulation, find

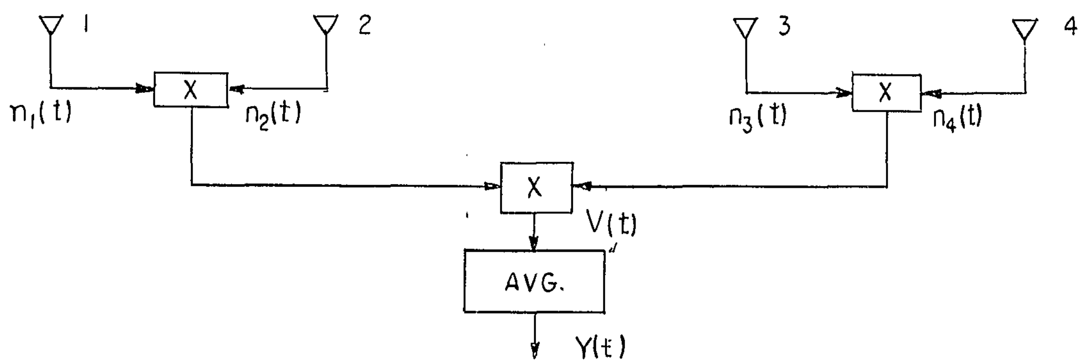
$$\psi_{12}(s) = R_n(0) \frac{\sin k(\frac{\pi Bd}{c})}{k(\frac{\pi Bd}{c})} \frac{\sin(\frac{\pi Bd}{c})}{(\frac{\pi Bd}{c})} \frac{\sin \pi Bs}{\pi Bs} \cos\omega_o s$$

$$= R_n(\tau) \frac{\sin \pi Bs}{\pi Bs} \cos\omega_o s$$

where $R_n(\tau) = R_n(0) \frac{\sin(\frac{\omega_o d}{c})}{(\frac{\omega_o d}{c})} \frac{\sin(\frac{\pi Bd}{c})}{(\frac{\pi Bd}{c})}$

Since in most cases k>>1, while d, at the most, is a few wavelengths, the last factor, $\sin(\frac{\pi Bd}{c})/(\frac{\pi Bd}{c})$, will have very little influence on the expression; and the cross-correlation coefficient can be considered to be a function of $\sin(\frac{\omega_o d}{c})/(\frac{\omega_o d}{c})$. It should be remembered that the approximation used to simplify the sine integrals in the original equation is not valid for small values of element spacing, d; and, in the event of close element spacings, calculation of the cross-correlation coefficient would require use of the exact equation.

Calculation of noise voltage and noise power present at the output of a simple nonlinear array will demonstrate the considerations which must enter into a specification of antenna element spacing. Take for example a four-element array:



The noise sources are assummed to be uniformly distributed over the surface of a unit sphere surrounding the array. Then the average value of output voltage, $\overline{Y(t)}$, becomes:

$$\overline{Y(t)} = \overline{V(t)} = \overline{n_1(t)n_2(t)n_3(t)n_4(t)}$$

$$= R_n(\tau_{12})R_n(\tau_{34}) + R_n(\tau_{13})R_n(\tau_{24}) + R_n(\tau_{14})R_n(\tau_{23})$$

Then, as shown in a previous section, the average value of output power is:

$$\overline{Y(t)^2} = \frac{2}{T}\int_0^T (1 - \frac{s}{T})\ \overline{V(t)V(t+s)}\ ds$$

But, with four elements,

$$\overline{V(t)V(t+s)} = \overline{n_1(t)n_2(t)n_3(t)n_4(t)n_1(t+s)n_2(t+s)n_3(t+s)n_4(t+s)}$$

And, when expressed as multiples of the correlation coefficients of the element noise voltages, this results in an equation with some 105 terms. However, its spacings are selected to make $\psi_{ij}(s) = 0$, $(i \neq j)$, then

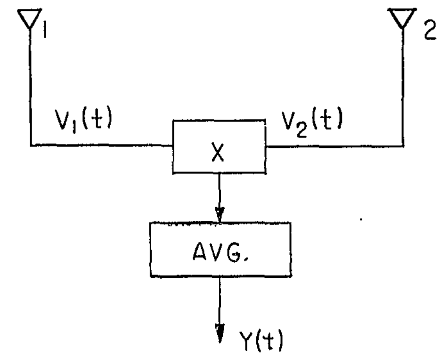
$$V(t)V(t+s) = [\psi_{11}(s)]^4 = R_n(0)^4 \left(\frac{\sin \pi Bs}{\pi Bs}\right)^4 \cos^4 \omega_0 s$$

and the output power can be readily evaluated. Spacings which give this result are multiples of a half-wavelength at the center frequency, f_o.

Since the cross-correlation coefficients can assume both positive and negative values, it is conceivable that some decrease in output noise power could be realized by appropriate changes in element spacings after a study of the complete expression for $\overline{V(t)V(t+s)}$ under the existing conditions of noise source distribution. However, the above method of considering all cross-correlation coefficients as zero gives a quick evaluation of output power which, by the very nature of the sin x/x function, is subject only to minor modification by more detailed considerations.

This development of the output noise power from a nonlinear array does not give the complete description of the operation of this type of array when it is used to detect a signal buried in a general noisy medium. Because of the multiplicative operations performed in the antenna circuitry, there will be an interaction between signal and noise which can appear as an undesired component of the output voltage.

This interaction will be shown by considering a simple two-element correlation array. Assume a single signal cource induces a voltage which is in phase on each element, and a uniform spherical distribution of noise sources induces undesired noise voltages on the elements.



Then

$$V_1(t) = S(t) + n_1(t)$$

$$V_2(t) = S(t) + n_2(t) = S(t) + n_1(t + \tau_{12}).$$

$$\overline{Y(t)} = \overline{S^2(t)} + \overline{S(t)[n_1(t) + n_2(t)]} + \overline{n_1(t)n_2(t)}$$

$$= \overline{S^2(t)} + R_n(\tau_{12}) = \overline{S^2(t)} + \psi_{12}(0)$$

If the element spacing is chosen (multiple of a half wavelength) to make $\psi_{12}(0) = 0$, then the voltage in the output circuit due to the signal is

$$\overline{Y(t)} = \overline{S^2(t)}.$$

Noise power, N, in the output circuit, can be determined from

$$N = \frac{2}{T} \int_0^T (1 - \frac{s}{T}) [\overline{Y(t)Y(t+s)} - \overline{Y(t)}^2] ds$$

$$= \frac{2}{T} \int_0^T (1 - \frac{s}{T}) \{\overline{S^2(t)S^2(t+s)} + \overline{S(t)S(t+s)} [\overline{n_1(t)n_1(t+s)}$$

$$+ \overline{n_2(t)n_2(t+s)}] + \overline{n_1(t)n_2(t)n_1(t+s)n_2(t+s)} - \overline{S^2(t)S^2(t)} \} ds$$

retaining only terms whose expectations are nonzero.

If the signal voltage induced on a single element is a simple sinusoid, $S = A \cos \omega t$, then the signal voltage in the output circuit is

$$\overline{Y(t)} = \frac{1}{2} A^2.$$

The signal power in the output circuit is

$$P = \frac{1}{4} A^4.$$

and the noise power is

$$N = \frac{2}{T}\int_0^T \left(1 - \frac{s}{T}\right)\left\{ \left(A^2 \cos \omega s\right)\left(R_n(0) \frac{\sin \pi Bs}{\pi Bs} \cos \omega s\right) + R_n(0)^2 \left(\frac{\sin \pi Bs}{\pi Bs} \cos \omega s\right)^2 \right\} ds$$

Since $f_0 >> B$, the ac component of the noise will be essentially zero in averaging times such that BT>1. So only the low frequency component of the noise power in the above equation need be considered.

$$N = \frac{2}{T}\int_0^T \left(1 - \frac{s}{T}\right)\left\{\frac{A^2 R_n(0)}{2} \frac{\sin \pi Bs}{\pi Bs} + \frac{R_n(0)^2}{2} \frac{\sin^2 \pi Bs}{(\pi Bs)^2}\right. ds$$

For large values of BT, this can be approximated by

$$N \approx \frac{A^2 Rn(0)}{2} \frac{1}{BT} + \frac{Rn(0)^2}{2} \frac{1}{BT}$$

So the noise power appears not only as a result of the noise sources alone (last term) but also as a result of the interaction or signal and noise voltages (middle term).

This total noise power resulting from the interaction of signal and noise voltages may well be greater than that caused by the noise sources alone. For this two-element nonlinear array and under conditions of unity signal-to-noise ratio on an individual element, the total noise power resulting from the cross multiplication is twice the noise power caused by the noise sources alone.

Nonlinear arrays of this same type (combining network composed of produce circuits only), but with greater numbers of elements, will have additional noise power terms resulting from this cross multiplication of signal and noise voltages. This is summarized in the following table. In this table, the level of multiplication refers to the number of times the voltage induced on an element undergoes multiplication before reaching the output circuit.

SIGNAL POWER AND NOISE POWER IN THE OUTPUT CIRCUIT OF A NONLINEAR ANTENNA ARRAY

Signal Voltage induced on each element $S = A \cos t$

Number of Elements	Level of Multiplication	Signal Power In Output CKT.	Noise Power ($BT \geq 1$)
1	1	$\frac{1}{2}A^2$	$\frac{1}{BT}[R_n(0)]$
2	1	$\frac{1}{4}A^4$	$\frac{1}{BT}\left[\frac{A^2R_n(0)}{2} + \frac{R_n(0)^2}{2}\right]$
4	2	$\frac{9}{64}A^8$	$\frac{1}{BT}\left[\frac{9}{16}A^6R_n(0) + \frac{30}{32}A^9R_n(0)^2 + \frac{9}{16}A^2R_n(0)^3 + \frac{1}{4}R_n(0)^4\right]$
6	3	$\frac{25}{256}A^{12}$	$\frac{1}{BT}\left[\frac{150}{256}A^{10}R_n(0) + \frac{390}{256}A^8R_n(0)^2 + \frac{420}{256}A^6R_n(0)^3 + \frac{40}{32}A^4R_n(0)^4 + \frac{575}{1024}A^2R_n(0)^5 + \frac{78}{160}R_n(0)^6\right]$

VI. NONLINEAR ANTENNAS AND CHANNEL CAPACITY

A direct comparison of the effect on channel capacity of the linear and nonlinear antennas is complicated by the change in dimensionality of the message space effected by the multiplicative and averaging processes. A comparison is possible, however, between the nonlinear antenna system and a linear antenna with an associated square law circuit. The signal-to-noise ratios in the outputs of these two systems can show the effectiveness of the nonlinear antenna system as related to the familiar linear antenna system.

Consider an antenna array of p elements, spaced so the random noise voltages induced on any two elements are uncorrelated. Then, if s is the voltage induced on each element by a signal cource in the main lobe, and n_i is the random noise voltage induced on the i-th element, the voltage from the linear array is

$$V_L = ps + \sum_{i=1}^{p} n_i .$$

After squaring and averaging

$$Y_L = \overline{V_L^2} = p^2\overline{s^2} + pR_n(0).$$

But if these p elements are split into two groups of p_1 and p_2 elements respectively ($p_1 + p_2 = p$), and the voltages of these two sub-arrays are multiplied and averaged, the output of this nonlinear array becomes:

$$Y_{NL} = \overline{(p_1 s + \sum_{i=1}^{p_1} n_i)(p_2 s + \sum_{j=1}^{p_2} n_j)} = p_1 p_2 \overline{s^2} .$$

The noise power in the output of each system will depend on the type of signal source. Two such sources will be examined: (1) a random signal source with a Gaussian distribution, and (2) a simple sinusoid, $s = A \cos \omega t$. Then, in terms of the signal-to-noise power ratio, $\frac{S}{N}$,

1. s Gaussian.

Linear Array $\left(\frac{S}{N}\right)_{out} = BT\left(\frac{S}{N}\right)^2_{in}\left[\frac{1}{\left(\frac{S}{N}\right)^2_{in} + \frac{2}{p}\left(\frac{S}{N}\right)_{in} + \frac{1}{p^2}}\right]$

Nonlinear Array $\left(\frac{S}{N}\right)_{out} = BT\left(\frac{S}{N}\right)^2_{in}\left[\frac{2p_1p_2}{2p_1p_2\left(\frac{S}{N}\right)^2_{in} + p\left(\frac{S}{N}\right) + 1}\right]$

when $p_1 = p_2 = \frac{p}{2}$

$$\left(\frac{S}{N}\right)_{out} = BT\left(\frac{S}{N}\right)^2_{in}\left[\frac{1}{\left(\frac{S}{N}\right)^2_{in} + \frac{2}{p}\left(\frac{S}{N}\right) + \frac{2}{p^2}}\right]$$

2. $s = A \cos \omega t$.

Linear Array $\left(\frac{S}{N}\right)_{out} = BT\left(\frac{S}{N}\right)^2_{in}\left[\frac{1}{\frac{1}{p^2} + \frac{2}{p}\left(\frac{S}{N}\right)_{in}}\right]$

Nonlinear Array $\left(\frac{S}{N}\right)_{out} = BT\left(\frac{S}{N}\right)^2_{in}\left[\frac{2p_1p_2}{1 + p\left(\frac{S}{N}\right)_{in}}\right]$

when $p_1 = p_2 = \frac{p}{2}$

$$\left(\frac{S}{N}\right)_{out} = BT\left(\frac{S}{N}\right)^2_{in}\left[\frac{1}{\frac{2}{p^2} + \frac{2}{p}\left(\frac{S}{N}\right)_{in}}\right]$$

where $\left(\frac{S}{N}\right)_{in}$ refers to the signal-to-noise power ratio induced on a single element of the array, and $\left(\frac{S}{N}\right)_{out}$ refers to the signal-to-noise power ratio at the output of the antenna array combining circuit.

When the signal-to-noise ratio and the number of elements are such that their product is large $\left[p\left(\frac{S}{N}\right)_{in} >> 1\right]$, the output signal-to-noise ratio is approximately the same for both arrays:

$$\left(\frac{S}{N}\right)_{out} \approx BT\left(\frac{S}{N}\right)_{in}\left(\frac{p}{2}\right).$$

In this case, the channel capacity would be expected to be the same for both arrays if the same number of elements, p, were used in each.

When the signal-to-noise ratio and the number of elements are such that their product is small $\left[p\left(\frac{S}{N}\right)_{in} << 1\right]$, the output signal-to-noise ratios become:

$$\text{Linear Array} \quad \left(\frac{S}{N}\right)_{out} \approx BT\left(\frac{S}{N}\right)^2_{in}\left(p^2\right)$$

$$\text{Nonlinear Array} \quad \left(\frac{S}{N}\right)_{out} \approx BT\left(\frac{S}{N}\right)^2_{in}\left(\frac{p^2}{2}\right)$$

Now the channel capacity of a nonlinear array is not so great as that of a linear array with the same number of elements, p.

This analysis has tended to ignore one very important aspect of the nonlinear array. The average output voltage of the linear array (Y_L) will contain a component proportional to the signal and a component proportional to the noise; the average output voltage of the nonlinear array (Y_{NL}) has no component which is independent of the signal. This emphasized the real difference between the linear and nonlinear arrays; the nonlinear array will generally be capable of a smaller channel capacity than the linear array, but in the final output voltage the effect of the random noise sources can be made arbitrarily small in the nonlinear array, while this is not possible in the linear array.

VII. DESIGN OF NONLINEAR ARRAYS

A. Basic Considerations

The preceeding analyses form the basis for certain conclusions regarding the employment of antenna arrays which utilize nonlinear processes in the combining network:

1. Random noise voltages induced on the antenna elements can be reduced to any desired level by proper element spacing and by averaging for a sufficiently long interval. Noise power is approximately inversely proportional to the averaging interval, BT.

2. Cross multiplication processes in the antenna circuitry will introduce additional noise terms resulting from interaction of signal and noise voltages, of different noise voltages, or of different signal voltages. These added noise terms attain their minimum values when the level of multiplication is limited to one.

3. When the level of multiplication is greater than one, the interaction of two or more independent signal sources will introduce components in the output voltage which cannot be reduced by averaging.

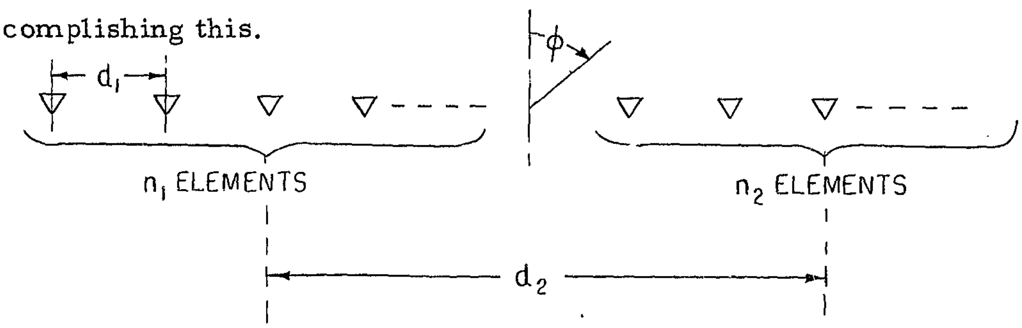
4. The examples of nonlinear antennas considered have demonstrated directional patterns with main lobes significantly narrower than those obtainable from linear arrays of the same size.

5. The nonlinear array is not always efficient in a general communication system, but offers particular advantages to a system in which narrow beamwidth is paramount.

These general conclusions will be the basis for the design of nonlinear antenna arrays. The difficulties which arise when the level of multiplication exceeds one outweigh any apparent increase in directivity which might be obtained. Therefore, the combining network will be a combination of addition and multiplication circuits, with the level of multiplication limited to one.

B. Products of Linear Arrays.[1]

If we consider a uniform array of n equally spaced isotropic elements, the simplest type of nonlinear antenna which can be constructed (with level of multiplication equal to one) is that in which the elements are formed into two linear arrays whose output voltages are then multiplied and averaged. There are, of course, a number of different combinations of the elements which can be realized in accomplishing this.



For two groups, containing n_1 and n_2 elements respectively, so that $n = n_1 + n_2$, and with spacings d_1 between elements and d_2 between group centers, the directional patterns can be written:

$$P_{n_1}(\psi) = \frac{\sin n_1(\psi/2)}{n_1 \sin(\psi/2)}$$

$$P_{n_2}(\psi) = \frac{\sin n_2(\psi/2)}{n_2 \sin(\psi/2)}$$

and
$$P_n(\psi) = \left(\frac{\sin n_1(\psi/2)}{n_1 \sin(\psi/2)}\right)\left(\frac{\sin n_2(\psi/2)}{n_2 \sin(\psi/2)}\right) \cos\left(\frac{d_2}{d_1}\psi\right)$$

[1] Welsby, V. G., and Tucker, D. G., "Multiplicative Receiving Arrays," Jour. British IRE, pp 369-382, June 1959

The two parameters controlling $P_n(\psi)$ are

1. the total number of elements, and
2. the number of elements in one of the groups.

$P_n(\psi)$ is equal to unity when $\psi = 0$. Each term in the expression for $P_n(\psi)$ will introduce zeros into the directional pattern. The first and second terms will have zeros equally spaced along the ψ-axis at intervals of $2\pi/n_1$ and $2\pi/n_2$ respectively. There are $[(n_1-1)+(n_2-1) = n-2]$ such zeros between $\psi=0$ and $\psi=2\pi$. Assuming there is no overlapping of the two groups, $d_2/d_1 = n/2$; the final term in the directional pattern, $\cos(d_2/d_1)\psi = \cos n\psi/2$, has n zeros in the interval between $\psi = 0$ and $\psi = 2\pi$.

It is evident then that a uniform nonlinear array with $n_1+n_2 = n$ elements will have a directional pattern with $2(n-1)$ zeros instead of the $(n-1)$ which would be available for a simple n-element linear array.

The $\cos n\psi/2$ factor in the directional pattern is the important innovation of this type of nonlinear antenna. This factor will cause the pattern to pass through zero for $\psi = \pi/n$, regardless of the manner of splitting the n elements into two groups. If these n-elements were utilized as a uniform linear array, the first zero would occur at $\psi = 2\pi/n$. Thus the beamwidth of the principal lobe of the nonlinear array is approximately $\frac{1}{2}$ that of a linear array of the same size.

Since the location of n-2 zeros will depend on the choice of n_1 and $n_2 = n - n_1$, a number of directional patterns are possible for any fixed total number of elements, n. Figure 1 compares the directional patterns which result when $n = 6$.

Variation in element spacing could influence all of the terms in the equation for the directional pattern of the array. Figures 2 and 3 demonstrate this effect, showing the directional patterns which result when the n_1 elements are at spacing d_1, while the n_2 elements are at spacing $2d_1$.

In these figures and in other investigations of the directional patterns, it is apparent that the $\cos(d_2/d_1)\psi$ term caused by

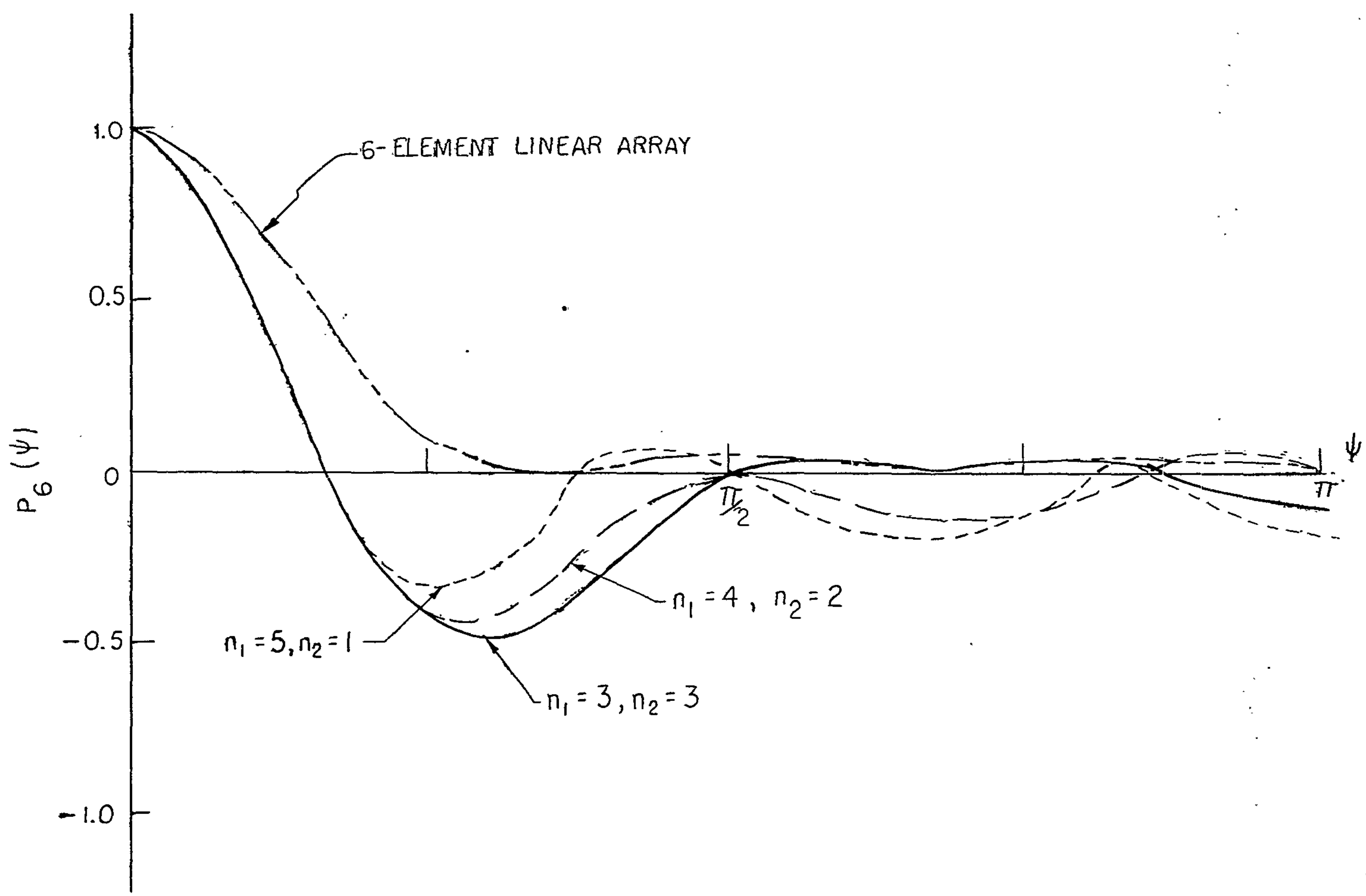


Figure 1: Directivity Patterns for a Six-Element Nonlinear Array

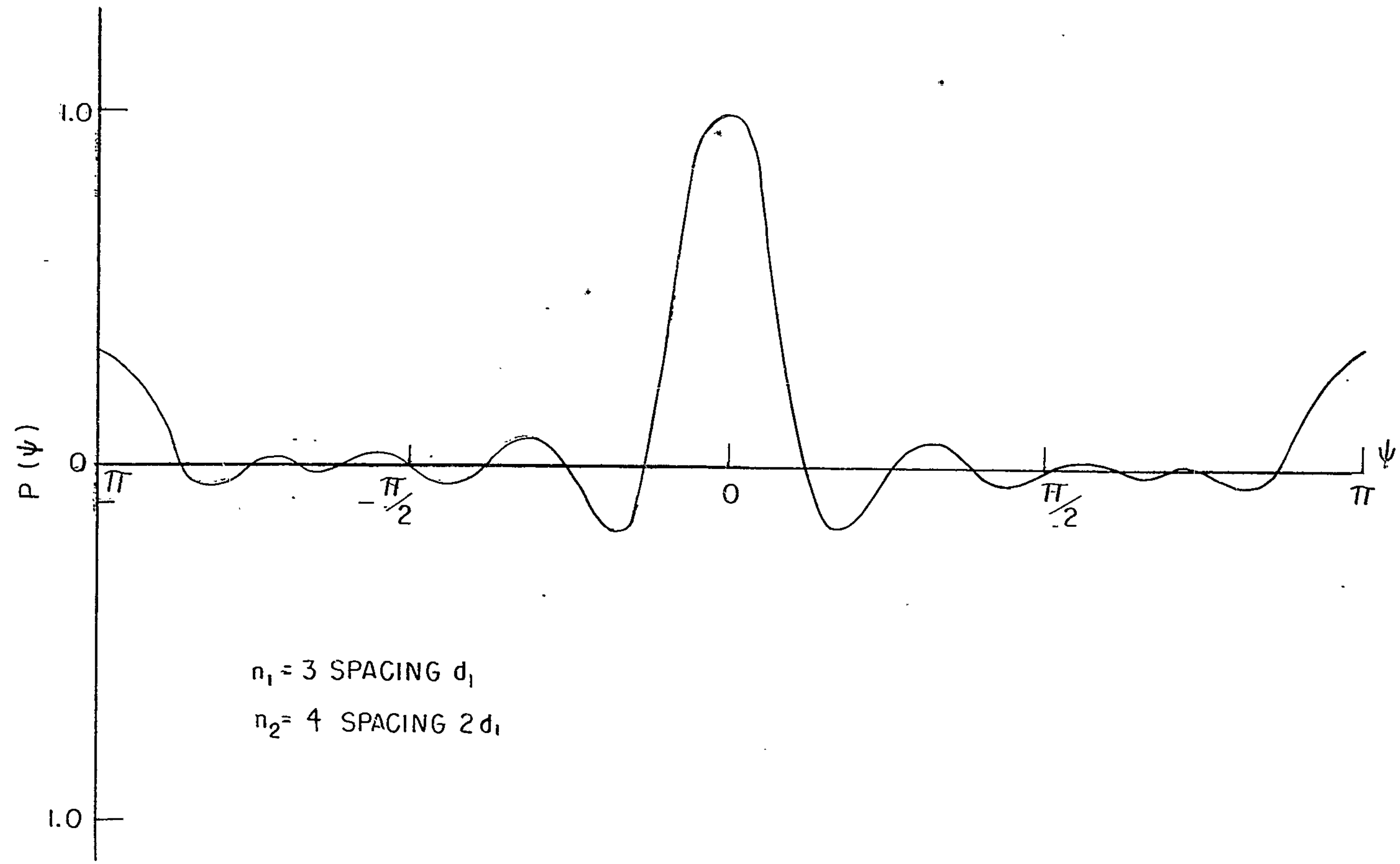


Figure 2: Directivity Pattern for Nonlinear Array with Nonuniform Spacing

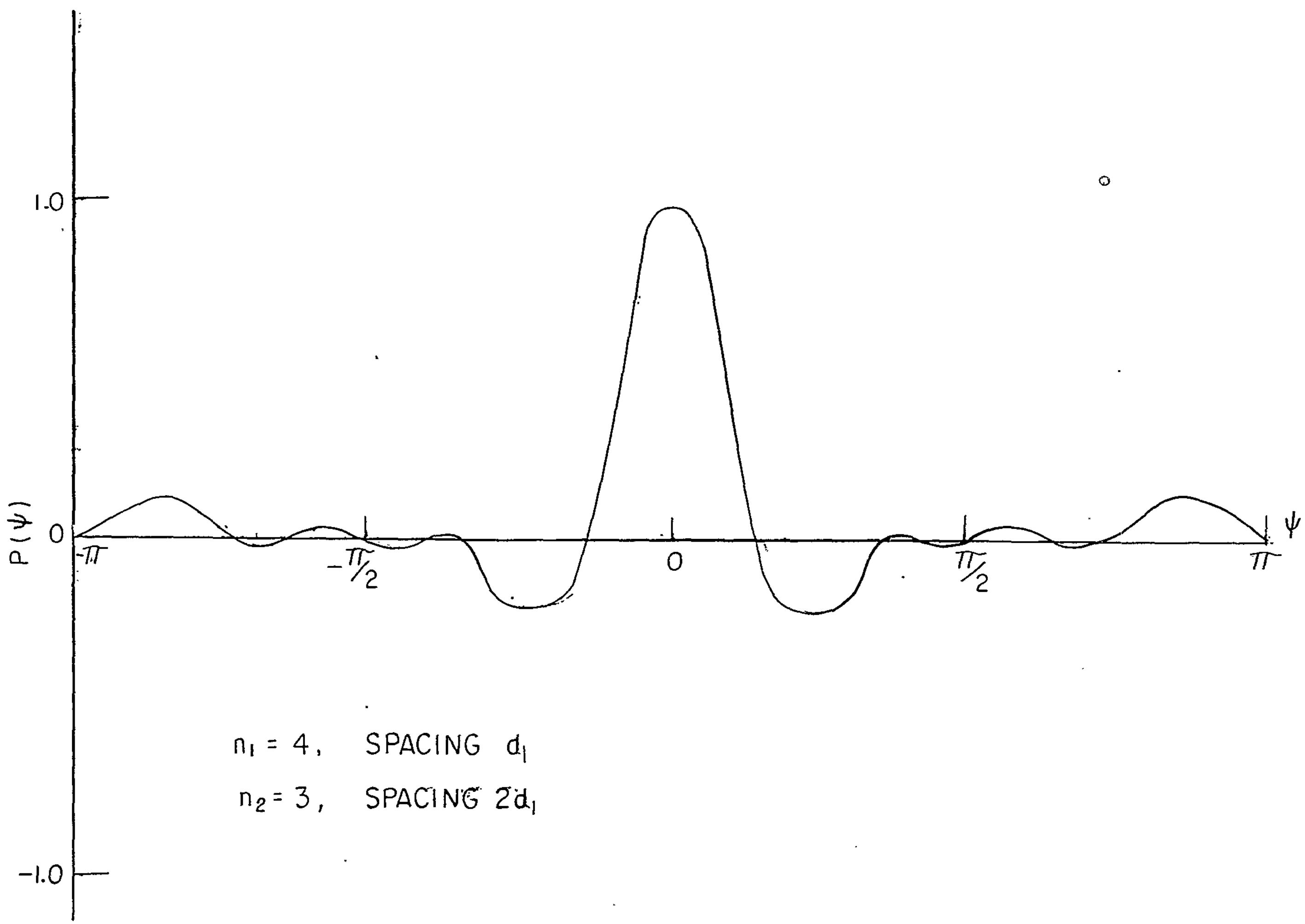
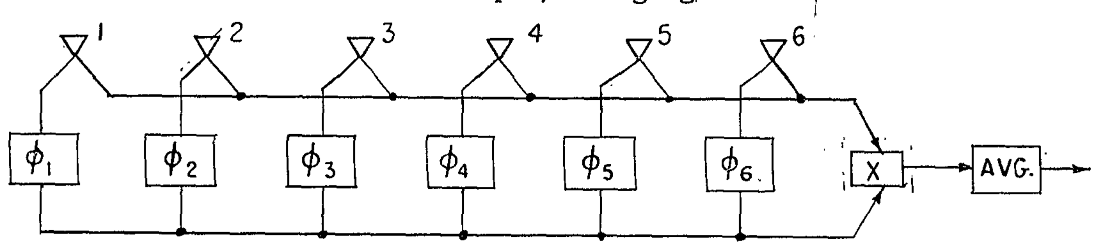


Figure 3: Directivity Pattern for Nonlinear Array with Nonuniform Spacing

separation of group centers has the greatest effect in narrowing the principal lobe.

C. Data Processing by Correlation Circuits.

Data processing by correlation circuits offers an attractive possibility for improving the presentation of the information available from an antenna array. Such a system could be based on the multiplication of the output voltage of one linear array by the output voltage of a second linear array in which a progressive phase shift has been introduced. For example, using again a total of six elements:



By providing a number of phase shifted circuits with the progressive phase shifts chosen to form directional patterns whose maxima occur at the zeros of the unshifted pattern or at the sidelobe maxima of the unshifted pattern, a number of such correlations processes can be carried out. By combining these correlation voltages into a single output voltage, an improved directional pattern is formed.

Figure 4 shows this resulting pattern for the six-element array. The unshifted pattern is given by

$$P_6(\psi) = \frac{\sin 6(\psi/2)}{6 \sin(\psi/2)} \quad .$$

The phase shifted patterns are given by

$$P_6'(\psi) = \frac{\sin 6\frac{1}{2}(\psi \pm \alpha)}{6 \sin \frac{1}{2}(\psi \pm \alpha)}$$

where $\alpha = \pi/6$, $\pi/3$, $\pi/2$, $2\pi/3$, $5\pi/6$, and π.

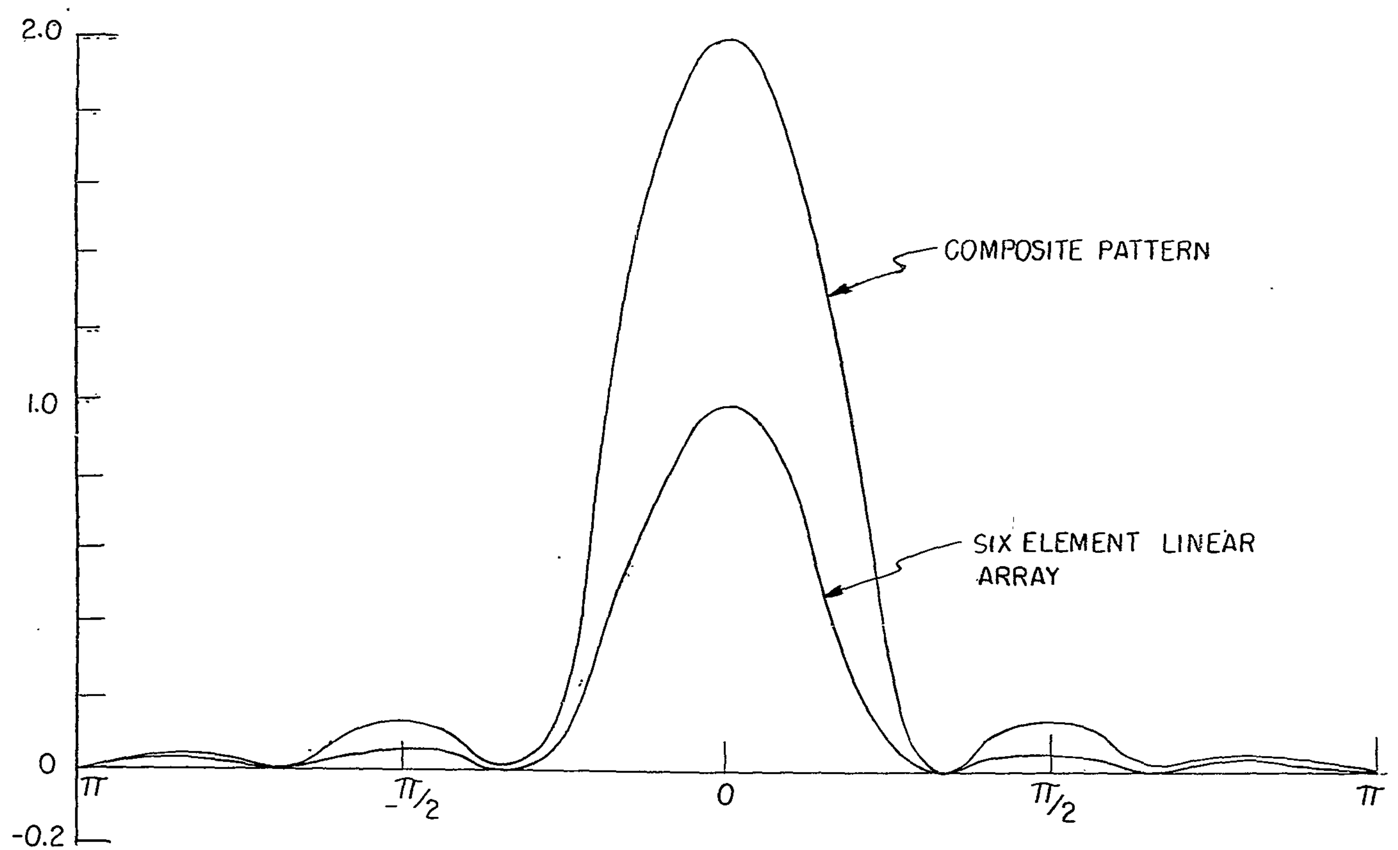


Figure 4: Directivity Pattern, Six-Element Array, Multiple Beam Correlation

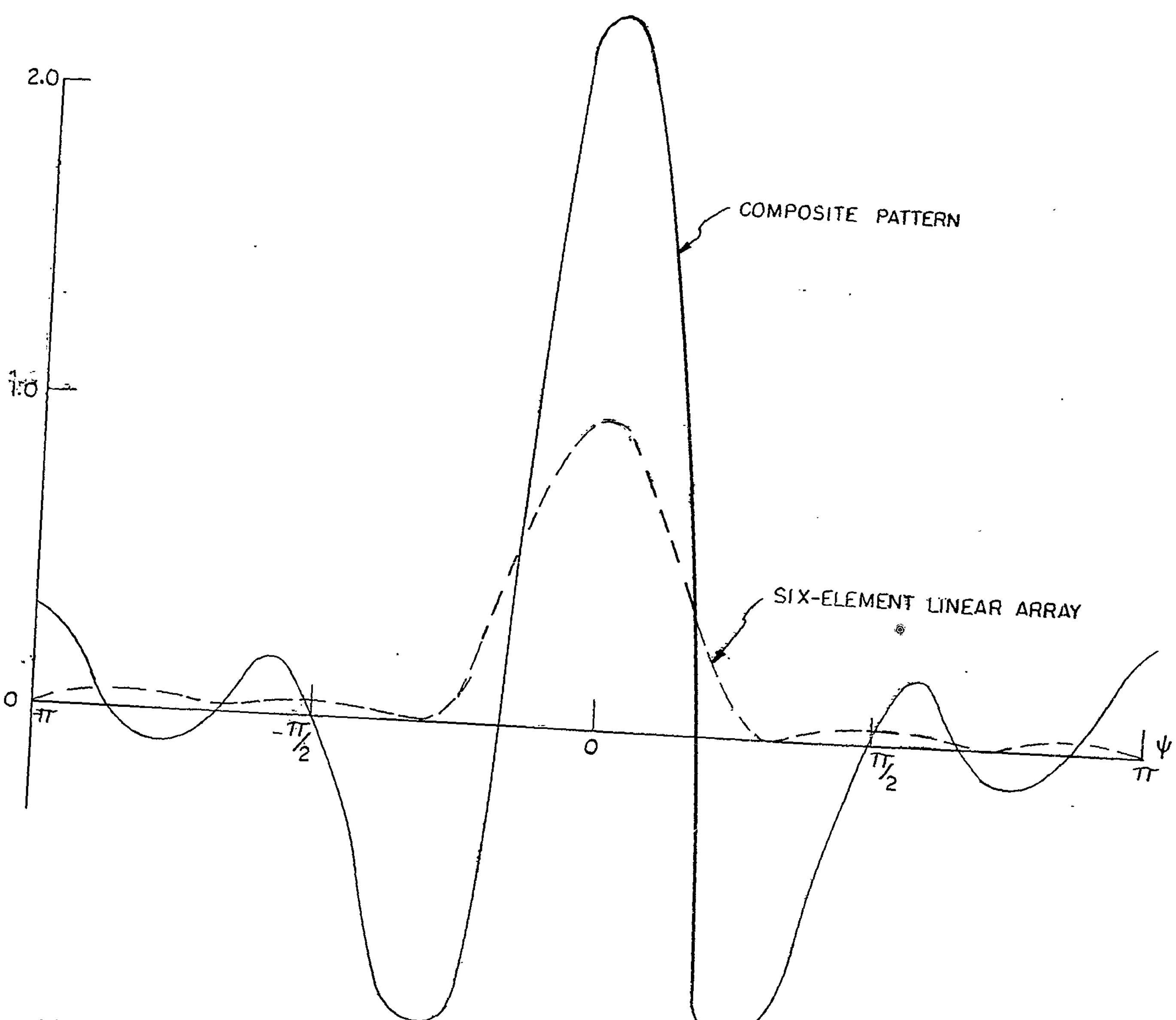
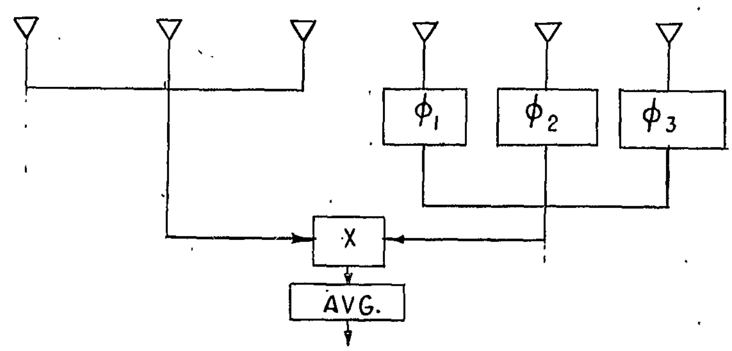


Figure 5: Directivity Pattern, Two Three-Element Arrays, Multiple Beam Correlation

By fully utilizing all of the phase shifted circuits connected to these six elements, it would be possible to provide a number of antenna patterns. with main beams displaced (equally in ψ) throughout 360°, all from a stationary array. This multiple-beam antenna is not new, but the improved beamwidth provided by the correlators permits a refinement in performance without increasing the overall length of the array.

Using the same six elements to provide unshifted as well as phase shifted voltages will not permit elimination of noise voltages by averaging. And it does not take advantage of the beam narrowing feature of the $\cos(d_2/d_1)\psi$ term present when the linear arrays are separated. Thus some improvement in performance would be expected from splitting the elements into two separated linear arrays, with progressive phase shifting applied to one of these resulting arrays. Figrue 5 shows the pattern which results when the six elements are formed into two uniformaly spaced three-element linear arrays.



$$P_6(\psi) = \frac{\sin 3(\psi/2)}{3\sin(\psi/2)} \quad \frac{\sin 3\frac{1}{2}(\psi \pm \alpha)}{3\sin\frac{1}{2}(\psi \pm \alpha)} \quad \cos(3\psi \pm \alpha)$$

where $\alpha = 0\pi/3$, $2\pi/3$, and π.

Appendix 1: THE PROBABILITY DISTRIBUTION FOR THE OUTPUT NOISE VOLTAGE OF A TWO-ELEMENT CORRELATION ARRAY.

A random distribution of noise sources surrounding a two-element antenna array will induce voltages which can be represented by stationary time series with Gaussian probability distributions. The noise voltages can thus be completely described statistically by specifying their cross- and auto- correlation functions. Each noise source will induce a voltage on one element which is identical to that induced on the other element, but with a time delay factor due to the finite time of travel of the wavefront from one element to the other. As shown previously, the cross- and auto-correlation functions of the total induced voltage can be obtained by summing the contributions of the individual sources. For a uniform distribution of independent noise sources and rectangular band limiting:

$$\psi_{11}(s) = 4\pi B\eta \frac{\sin \pi Bs}{\pi Bs} \cos \omega_o s = R_n(0) \frac{\sin \pi Bs}{\pi Bs} \cos \omega_o s$$

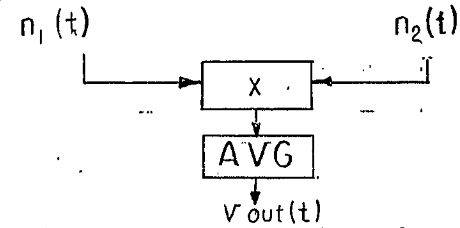
$$\psi_{12}(s) = R_n(0) \frac{\sin k(\frac{\pi Bd}{c})}{k(\frac{\pi Bd}{c})} \frac{\sin(\frac{\pi Bd}{c})}{(\frac{\pi Bd}{c})} \frac{\sin \pi Bs}{\pi Bs} \cos \omega_o s$$

$$= R_n(\tau) \frac{\sin \pi Bs}{\pi Bs} \cos \omega_o s$$

where:

1) noise sources in element of angle $d\Omega$ produce noise power $\eta d\Omega$ watts/cps;
2) B is bandwidth, ω_o is center frequency;
3) $k = \frac{\omega_o}{\pi B}$, and d is element spacing.

The elements of the system can be represented as:



When $n_1(t) = n_2(t)$, the circuit is that of a square law detector; the statistics of this system have been studied by several authors.[1,2] Emerson has shown that the probability distribution of the output voltage of the square law detector is determined by the eigenvalues of a linear, homogeneous, integral equation whose kernel is a function only of the correlation function of the input voltage and the impulse response of the circuit employed as an averager.

Using this general method Lampard[3] has obtained an explicit solution for correlated input voltages, but only for the case of no postmultiplier filtering.

Although it appears that there is no explicit solution to this general antenna noise problem when postmultiplier filtering is employed, the resulting distribution can be represented by an infinite series whose significant terms can be calculated. If these input noise voltages have bandlimited Gaussian distributions, the output from the averaging circuit can be expressed in terms of an infinite series and displayed graphically in a manner similar to that used by Emerson.

The output voltage of the circuit is written as a convolution

$$v_o(t) = \int_0^t n_1(t-u)n_2(t-u)h(u)\, du$$

---

[1] Emerson, R. C., "First Probability Densities for Receivers with Square Law Detectors", Jour. Appl. Phys., vol. 24, no. 9, pp 1168-1176, Sept. 1953.

[2] Meyer, M. A. and Middleton, D., "On the Distribution of Signals and Noise After Rectification and Filtering," Jour. Appl. Phys., vol. 25, pp 1037-1052, Aug. 1954,

[3] Lampard, D. G., "The Probability Distribution for the Filtered Output of a Multiplier Whose Inputs are Correlated, Stationary, Gaussian Time-Series," IRE Trans., vol IT-2, no. 1, pp 4-11, March 1956.

where h(u) is the impulse response of the averaging circuit. Since, for this antenna situation, $n_1(t)$ differs from $n_2(t)$ only by the time delay factor τ, the output can be written

$$v_o(t) = \int_0^t n_1(t-u)n_1(t + \tau - u)h(u)\, du.$$

The moments of the output distribution can be determined directly:

$$\overline{v_o(t)} = \int_0^t \overline{n_1(t-u)n_1(t + \tau - u)h(u)}\, du$$

For this perfect averaging circuit

$$h(u) = \begin{cases} \frac{1}{t}, & 0 \le u \le t \\ 0, & \text{elsewhere} \end{cases}$$

Then

$$K_1 = \overline{v_o(t)} = \frac{1}{t}\int_0^t \psi_{12}(0)\ du$$

where $\psi_{ij}(s) = \overline{n_i(t)n_j(t+s)}$ are the correlation functions for the bandlimited noise voltages.

$$K_2 = \overline{v_o(t)^2} = \frac{1}{t^2} \int_0^t \int_0^t \overline{n_1(t-u)n_2(t-u)n_1(t-v)n_2(t-v)}\ du\ dv$$

$$= \frac{1}{t^2} \int_0^t \int_0^t [\psi_{12}(0)^2 + \psi_{11}(u-v)^2 + \psi_{12}(u-v)^2]\ du\ dv.$$

The variance is $\sigma^2 = K_2 - K_1^2$.

A general expression for the moments could be written in the form of an integrated integral

$$K_i = \overline{v_o(t)^i} = \frac{1}{t^i} \int_0^t \cdots \int_0^t \overline{n_1(t-x_1)n_2(t-x_1) - - - n_1(t-x_i)n_2(t-x_i)}\, dx_1 - - - dx_i .$$

The output probability distribution can now be determined in terms of a Gram-Charlier series, type A.[4, 5] If we introduce the normalized variable

$$y = \frac{v_o(t) - \overline{v_o(t)}}{\sigma}$$

then a convenient expansion for the probability distribution of y is given by the series grouped according to Edgeworth:

$$p(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2} \{1 + C_3H_3(y) + [C_4H_4(y) + C_6H_6(y)] + - - - -\} ,$$

where $H_n(x)$ is the Hermite polynomial of degree n defined by

$$\frac{d^n}{dx^n} (e^{-\frac{1}{2}x^2}) = (-1)^n H_n(x)\, e^{-\frac{1}{2}x^2} ,$$

where

$$C_i = \frac{\mu_i}{(i\,!)\sigma^i}$$

and μ_i is the i-th central moment, obtained from K_i, the i-th moment.

To illustrate the change in the probability distribution of the output voltage with increasing averaging time, let us consider two isotropic antenna elements in a uniformly distributed noisy medium with an element spacing $d = \frac{\lambda}{4}$. For this element spacing, $R_n(\tau) = \frac{2}{\pi} R_n(0)$.

With no postmultiplier filtering the distribution is given explicitly by Lampard:

$$p(v_o) = \frac{1}{\pi}[\psi_{11}(0)^2 - \psi_{12}(0)^2]^{-\frac{1}{2}} \exp\{v_o \frac{\psi_{12}(0)}{\psi_{11}(0)^2 - \psi_{12}(0)^2}\} K_o \{|v_o| \frac{\psi_{11}(0)}{\psi_{12}(0)^2 - \psi_{12}(0)^2}\},$$

where $K_o(x)$ is the modified Bessel function of the second kind and zero order.

With postmultiplier averaging included, the equations for the first three central moments of the distribution are:

$$\mu_1 = \overline{v_o(t)} = \frac{1}{t}\int_0^t R_n(\tau)\, du = R_n(\tau) = \frac{2}{\pi} R_n(0) .$$

$$\mu_2 = \sigma^2 = \frac{1}{t^2} \int_0^t \int_0^t \{[R_n(0)^2 + R_n(\tau)^2] \frac{\sin^2 \pi B(u-v)}{[\pi B(u-v)]^2} \cos^2 \omega_o(u-v)\}\, du\, dv .$$

$$\mu_3 = \frac{1}{t^3} \int_0^t \int_0^t \int_0^t \{[6R_n(0)^2 R_n(\tau) + 2R_n(2)^3] \frac{\sin \pi B(u-v)}{\pi B(u-v)} \frac{\sin \pi B(v-\omega)}{\pi B(v-\omega)}$$

$$\frac{\sin \pi B(\omega-u)}{\pi B(\omega-u)} \cos \omega_o(u-v) \cos \omega_o(v-\omega) \cos \omega_o(\omega-u)\}\, du\, dv\, d\omega .$$

The output probability density functions are now expressed in terms of the Gram-Charlier series. They have been plotted in Figure 6 for selected values of the bandwidth-averaging time product, Bt. The coefficients have been obtained by neglecting the terms in the $\cos \omega_o(u-v)$ products which represent the high frequency residue. Since in a typical circuit $\omega_o >> B$, they are vanishingly small for values of Bt considered here. Actual solution of the integrals to obtain the series coefficients was carried out on a Bendix G-15 digital computer.

For the case Bt = 0 (no averaging) the curve is exact, and the effect of all frequencies is considered.

For Bt = 1, Bt = 2, and Bt = 4, the convergence if fairly rapid (three terms of the series required) for graphical accuracy in regions near the mean. Accuracy for the extremes is not claimed, but would require additional terms in the series.

It will be noted that the tendency toward normality with increasing Bt (increasing averaging time) is pronounced. This is in general agreement with the results observed by Emerson for the square law detector and would be expected from the general behavior of integrated noise derivable from the Central Limit Theorem.

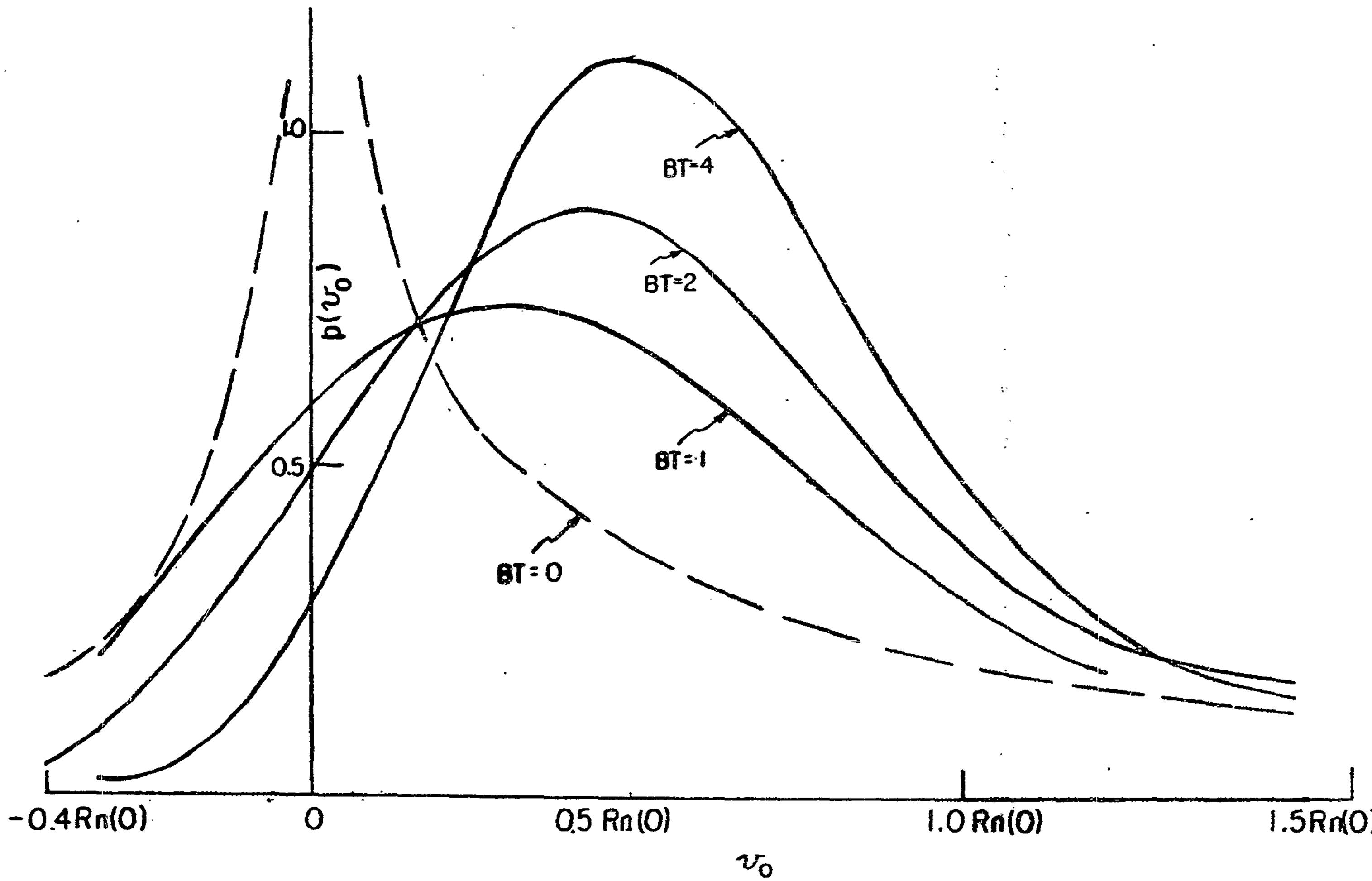


Figure 6: **Probability Density Functions for the Output Noise Voltage of a Two-Element Correlation Array, for Various Averaging Times. Element Spacing d = 1/4 λ**

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